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ANALYTICAL STUDIES FOR THE CRITICAL LINE AND CRITICAL ENDPOINT OF THE ELECTROWEAK PHASE TRANSITION^a

H. MEYER-ORTMANN

*Fachbereich 8 Physik, Bergische Universität Wuppertal, Gaußstr.20,
D-42097 Wuppertal, Germany*

E-mail: ortmanns@theorie.physik.uni-wuppertal.de

T. REISZ

*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16,
D-69120 Heidelberg, Germany*

E-mail: reisz@thphys.uni-heidelberg.de

We outline two approaches for studying the electroweak phase transition in the framework of the four-dimensional SU(2) Higgs model on a lattice. The first one is based on a combination of variational estimates for the free energy and a solution of the resulting gap equations by means of dynamical linked cluster expansions. In the second approach we first indicate the derivation of an effective scalar field theory upon integration over the gauge fields. The phase structure of the resulting scalar theory is then investigated by means of generalized linked cluster expansions. We present results for the critical line and the critical endpoint.

1 Introduction

We determine the critical line and critical endpoint of the electroweak transition in the framework of the SU(2) Higgs model on the lattice in four dimensions. Usually the electroweak transition in the SU(2) Higgs model is studied with Monte Carlo simulations either in the four-dimensional model^{1, 2} or -after dimensional reduction- in an effective three-dimensional model of the same form³. The time extensions in such Monte Carlo simulations is restricted to small values such as $N_\tau = 2, 3$ unless one uses asymmetric lattices, and λ is of $O(10^{-4})$. In our approach we first want to integrate out the gauge field degrees of freedom and then study the phase structure of the resulting effective scalar model with *Linked Cluster Expansions* (LCEs) that have been

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developed to a powerful tool by extending the expansions to a high order in the expansion parameter and to a finite volume^{4, 5}.

2 Critical Line from Variational Estimates and DLCEs

In the first part we describe an analytic estimate of the critical line $\kappa_c(\lambda)$ for a given gauge coupling β . The estimate is based on a combination of variational estimates for the free energy of the SU(2) Higgs model in 4D and *Dynamical Linked Cluster Expansions* (DLCEs) for correlators in the set of variational equations. Already in this first estimate the localization of the critical line agrees quite well with high precision Monte Carlo results, since even this estimate became rather complex after all.

Variational estimates for the free energy are based on the convexity of the exp-function and the positivity of the measure. Under these conditions we obtain an inequality between partition functions which is of the form

$$\exp(-Vf) \equiv Z \geq Z_{VE} \cdot \exp < -(S - S_{VE}(\zeta)) >_{VE} \equiv \exp -V\tilde{f}(\zeta) \quad (1)$$

with the following notations. V is the D-dimensional volume, f is the free energy density of the system described by the partition function Z , in our case Z is the partition function of the SU(2) Higgs model in D=4, S_{VE} (Z_{VE}) is the auxiliary action (partition function), respectively, that depends on a generic set of variational parameters ζ , and \tilde{f} is the trial free energy density which should be optimized as a function of ζ in order to minimize $\tilde{f} - f \geq 0$. The generic set of variational equations is then given as

$$\partial_\zeta \tilde{f}(\zeta) |_{\zeta=\tilde{\zeta}} = 0 \quad \partial_\zeta^2 \tilde{f}(\zeta) |_{\zeta=\tilde{\zeta}} > 0 \quad (2)$$

An equality sign in the second inequality determines the critical temperature T_c (coupling κ_c) in case of a second order transition and gives a lower (upper) bound on T_c , (κ_c) in case of a first order transition.

Next we have to find a good choice for S_{VE} in case of the SU(2) Higgs model. Since we know that the three-dimensional spatial part of the gauge-Higgs interaction is supposed to contain the non-perturbative degrees of freedom that drive the Higgs transition, in our final choice for S_{VE} , we treat the physics along three-dimensional spatial hypersurfaces as accurately as possible, but implement a factorization along the time direction so that

$$Z_{VE}(\zeta_{link}, \zeta_{cube}, \xi) = Z_{cube}^{L_0}(\zeta_{cube}, \xi) \cdot Z_{link}^{L_0 V_3}(\zeta_{link}) \quad (3)$$

in which Z_{link} is an exactly solvable one-link partition function depending on a variational parameter ζ_{link} . L_0 is the extension in time direction, V_3 the

3-dimensional volume. The action of the partition function Z_{cube} for the 3-D hypersurfaces is given as $S_{cube} = S$ with

$$S = - \sum_{x \in \Lambda_3} \left[2 (\xi \text{Tr} \Phi(x) + \zeta_{cube} \text{Tr} U(x; \mu)) + \kappa \sum_{\mu=1}^3 \text{Tr} (\Phi^\dagger(x) U(x; \mu) \Phi(x + \mu)) \right] \quad (4)$$

depending on variational parameters ζ_{cube} and ξ . Note that in contrast to usual LCEs the hopping term in (4) does depend on U , and U has its own dynamics governed by the second term in S_{cube} . Thus a Taylor expansion of $\ln Z_{cube}$ in powers of κ about $\kappa = 0$ will lead to a more general type of expansion, since the familiar LCE only apply to frozen U -dynamics. We will need such expansions for derived quantities from $\ln Z_{cube}$ that occur in the set of variational equations. The detailed form of the variational equations is given in ⁶. Here we only state that they depend on connected n -point functions in Φ s and U s up to $n = 6$, evaluated w.r.t. $\langle \cdot \rangle_{cube}$. It is these expectation values that we evaluate as power series in κ with DLCEs that have been introduced and systematized in ⁶. The number of graphs that contribute up to and including $O(\kappa^4)$ is of the order of several hundred. Finally we solve the variational equations for ζ_{link} , ζ_{cube} , ξ and κ_c as a power series in κ .

As results for κ_c we obtain for example $\kappa_c = 0.1282(1)$ ($0.1281(1)$) for $\lambda = 5 \cdot 10^{-4}$ ($5.1 \cdot 10^{-4}$), and $\beta = 8$ as compared to $\kappa_c = 0.12887(1)$ ($0.12852(2)$) from ⁷, respectively. The good quantitative agreement of low order DLCEs (including $O(\kappa^4)$) with high precision Monte Carlo results appears less surprising in view of the number of graphs that contribute to the series up to that order.

3 Critical Endpoint of the Electroweak Phase Transition

Starting from an $SU(2)$ -Higgs model in $4D$ we have derived⁸ an effective scalar model in the following steps. First we absorb the angular part of the $U(2)$ Higgs field in a gauge transformation. The remaining Higgs degrees of freedom are then given by scalar fields $\rho(x)$, $\rho(x) \geq 0$, so that $\text{Tr} \Phi^\dagger \Phi = \rho^2$. Integrating upon the U -dependent part of the original action leads to an effective action $W(\rho)$. $W(\rho)$ is the free energy of the gauge fields in a scalar background field. Since we are interested in an effective model that describes the phase structure in the vicinity of the critical endpoint with long range correlations, we perform a small momentum expansion of gauge field correlators about (lattice) momentum $\hat{p} = 0$ that appear as coefficients in $W(\rho)$. It is justified if $p/m_{glue} \ll 1$ and $p/T_c \ll 1$ (in a continuum language) with m_{glue} denoting the mass gap in the pure gauge sector. The small \hat{p} -expansion of the gauge field correlators induces an expansion of the action for the scalar fields with leading ultralocal

and local terms. Non-local terms are suppressed with higher momenta \hat{p} . The $\hat{p} = 0$ term of $W(\rho)$, $W^{(\circ)}$, is calculated in a Monte Carlo simulation for a pure gauge theory in a constant background field $r \equiv \rho(x)$ ⁸. The Monte Carlo results have been compared with results from a small field and large field expansion of W in terms of ρ . It turns out that the final value for the critical coupling κ_c sensitively depends on the shape of $W^{(\circ)}(\kappa r^2)$ for small values of κr^2 , but is rather insensitive to the precise form for large arguments. Thus we improve on the small field range by including contributions of the order \hat{p}^2 to the first term in a small field expansion, but neglect \hat{p}^2 and higher order terms of a large ρ -expansion, the leading contribution of which is absorbed in $W^{(\circ)}(\kappa r^2)$. The final form of the effective scalar model $S_{eff}(z)$ in terms of scalar fields $z \equiv z(x) = \rho^2(x)$ is then given by

$$S_{eff}(z) = \sum_{x \in \Lambda} \left(z + \lambda(z-1)^2 + W^{(\circ)}(\kappa z; \beta) + \frac{D\kappa^2}{2} z^2 - \frac{\kappa^2}{2} \sum_{\mu} z(x) z(x+\hat{\mu}) \right). \quad (5)$$

Although this model looks at most quadratic in z , it is non-Gaussian because of the term $W^{(\circ)}$ resulting from the Monte Carlo determination of the $\hat{p} = 0$ -part of the effective action W and $z \geq 0$. For given β , $W^{(\circ)} = W^{(\circ)}(\kappa z)$ is known in the form of a table for $\simeq 500$ values of $\kappa z \in [0, 25]$. Note that the model is no longer $Z(2)$ symmetric as the original $SU(2)$ Higgs model was in terms of Φ . The $Z(2)$ symmetry will only effectively be restored at the critical endpoint. The missing $Z(2)$ symmetry is also responsible for the generalization of LCEs to LCEs in an "external field"⁹. In terms of z the hopping term has its usual form, but it should be further noticed that now also two parts of the ultralocal part of the action do depend on κ . It is possible to account for this additional κ -dependence by a renormalization of the ultralocal vertices in the LCE graphs: the former constant coefficients in the series for the susceptibilities now are expanded themselves as power series in κ . The phase structure of this model is next studied with an effective potential with coefficients calculated to a high order in the expansion parameter of the series.

3.1 λ_c from an Effective Potential with LCEs

We approximate the partition function $Z(J)$ of the original $SU(2)$ Higgs model in 4D by $Z(J) = \int \mathcal{D}z \exp -S_{eff}(z) + J \cdot z$. The effective action $\Gamma(z)$ is then derived in the standard way by a Legendre transformation of $\ln Z(J)$ with $z \equiv d \ln Z(J) / dJ$. Evaluating $\Gamma(z)$ for constant field configurations z (corresponding to constant currents J), we obtain the effective potential V_{eff} .

Next we express the coefficients of V_{eff} in terms of quantities that are directly available in a high order linked cluster expansion. These are the sus-

ceptibilities that are 1PI in the LCE- graphical sense and are denoted by χ_n^{1PI} . The χ_n^{1PI} , $n = 2, \dots, 6$, are calculated up to and including $O(\kappa^{16})$. If we expand the effective potential in fluctuations about the vacuum expectation value $\tilde{z} := d \ln Z(J)/dJ|_{J=0}$, the qualitative form looks like

$$V_{eff}(x) = a_2 x^2 + a_3 x^3 + a_4 x^4 + O(x^5) \quad (6)$$

with $x = z - \tilde{z}$ and coefficients a_i , $i = 2, 3, 4$ that depend on β , λ , κ in a very implicit way. The phase structure of the effective scalar model with action S_{eff} is derived by scanning the sign of $D := (3a_3/8a_4)^2 - (a_2/2a_4)$ as a function of κ . A first order transition is indicated by $D > 0$ for a certain range of κ s between the occurrence and disappearance of the metastable second minimum in the symmetric and broken phase, respectively. In particular the critical endpoint (λ_c, κ_c) of the first order transition line for fixed β shows up as $D = 0$ and $a_2 = 0 = a_3$, or, in terms of the χ^{1PI} s, $4\kappa^2 \chi_2^{1PI}(\kappa, \lambda; \beta) = 1$ and $\chi_3^{1PI}(\kappa, \lambda; \beta) = 0$. If we extrapolate the results for $\lambda_c(L)$, $\kappa_c(L)$, $L = 7, \dots, 16$, to infinite order of truncation, $\lambda_c(L) = \lambda_c(\infty) + \text{const } 1/L$, we obtain $\lambda_c(\infty) = 0.0032(1)$ and, analogously, $\kappa_c(\infty) = 0.1447(1)$. An inclusion of χ_5^{1PI} and χ_6^{1PI} -terms in the effective potential shows that the neglected coefficients in (6) are suppressed by more than an order of magnitude.

4 Outlook

Feasible extensions in our framework include a calculation for the gauge coupling $\beta = 10$, larger extensions in time or larger values of λ . So far the LCE-programs were run on a SUN-workstation. Applied to the deconfinement transition of QCD with dynamical fermions, the first approach would lead to a localization of the line of critical hopping parameters, whereas a derivation of an effective scalar model (in terms of quark condensates) along the lines of the second approach applies to the chiral transition of QCD. Finally we remark that DLCEs have a much wider range of applications than it was indicated here. They include spin glasses, partially annealed neural networks, or diluted Ising models and are independent of any variational approach.

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